# M.Sc. (Mathematics)
## Courses and Syllabus

School of Mathematics

THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY
(Deemed to be University)

2022

**Name of Program:** Master of Science (Mathematics)

**Introduction** Mathematics programme is of utmost value to the aspiring graduate students with Mathematics background. Study of mathematics enables students to take up a variety of careers ranging from academic research, education, development, and quantitative applications in industry. The course provides students with ability to apply analytical and theoretical skills to model and solve mathematical problems. This programme has been introduced due to the need for sophisticated mathematics for modern scientific investigations and technological developments. The curriculum is designed to provide students with in depth pure as well applied mathematics so that a student become competent to take challenges in Mathematics at National and International levels.

**Nature:** Full time/ Part time/ Correspondence: Full Time.

**Duration:** Two Years (4 Semesters).

**Eligibility Criteria and Admission Procedure:** The candidates seeking admission to M.Sc. (Mathematics) must have a three year Bachelor degree with Mathematics as a major subject. The qualifying degree must be from a recognized University by the University Grants Commission with minimum duration of three years. The candidate must have secured at least 60% (55% for SC/ST) marks in qualifying degree. Admissions shall be made by merit which will be made by combining the percentages of marks obtained in 10th, 12th and graduation level. The degree marks will be considered up to second year/four semesters.

**Number of Seats: 20.**

**Program Educational Objective:** The objectives of the M.Sc. (Mathematics) program are to:

1. develop skills required for sound analytical and practical knowledge to pursue careers in Industries, Banks, Insurance, Educational, and Research Institute.
2. prepare students to qualify various national and international competitive examinations.
3. develop mathematical thinking encompassing logical reasoning, generalization, abstraction, and formal proof.

**Program Outcomes:** At the end of the program, the students will be able to:

1. acquire the knowledge and understanding of pure and applied mathematics and communicate mathematics effectively.
2. innovate, invent and solve complex mathematical problems using the knowledge of pure and applied mathematics.
3. pursue research career in mathematics and inter-disciplinary fields.
4. have the ability to assess and interpret complex situation, enabling them to choose successful career in education and industry.

# M.Sc. Mathematics : Courses[1]

## First Semester

| S.No. | Course No. | Course Name | L | T | P | Credits |
|---|---|---|---|---|---|---|
| 1. | PMA107 | Real Analysis | 3 | 1 | 0 | 3.5 |
| 2. | PMA108 | Algebra I | 3 | 1 | 0 | 3.5 |
| 3. | PMA109 | Ordinary Differential Equations | 3 | 1 | 0 | 3.5 |
| 4. | PMA110 | Mechanics | 3 | 1 | 0 | 3.5 |
| 5. | PMA111 | Computer Programming | 3 | 0 | 2 | 4.0 |
| | | **Total Credits** | | | | 18.0 |

## Second Semester

| S.No. | Course No. | Course Name | L | T | P | Credits |
|---|---|---|---|---|---|---|
| 1. | PMA204 | Measure Theory and Integration | 3 | 1 | 0 | 3.5 |
| 2. | PMA205 | Algebra II | 3 | 1 | 0 | 3.5 |
| 3. | PMA206 | Partial Differential Equations | 3 | 1 | 0 | 3.5 |
| 4. | PMA207 | Complex Analysis | 3 | 1 | 0 | 3.5 |
| 5. | PMA208 | Numerical Analysis | 3 | 1 | 2 | 4.5 |
| | | **Total Credits** | | | | 18.5 |

## Third Semester

| S.No. | Course No. | Course Name | L | T | P | Credits |
|---|---|---|---|---|---|---|
| 1. | PMA301 | Functional Analysis | 3 | 1 | 0 | 3.5 |
| 2. | PMA302 | Topology | 3 | 1 | 0 | 3.5 |
| 3. | PMA303 | Probability and Statistics | 3 | 1 | 2 | 4.5 |
| 4. | PMA304 | Mathematical Programming | 3 | 1 | 2 | 4.5 |
| 5. | | Elective I | | | | 4 |
| | | **Total Credits** | | | | 20 |

## Fourth Semester

| S.No. | Course No. | Course Name | L | T | P | Credits |
|---|---|---|---|---|---|---|
| 1. | PMA401 | Number Theory | 3 | 1 | 0 | 3.5 |
| 2. | PMA402 | Mathematical Methods | 3 | 1 | 0 | 3.5 |
| 3. | | Elective II | | | | 4 |
| 4. | PMA491 | Dissertation | | | | 10 |
| | | **Total Credits** | | | | 21 |

**Total Credits: 77.5**

[1] Approved in 107th Senate meeting held on June 16, 2022 (Revised).

# List of Electives

## Elective I [4 Credits]

| S.No. | Course No. | Course Name | L | T | P |
|---|---|---|---|---|---|
| 1. | PMA331 | Numerical Methods for Partial Differential Equations | 3 | 0 | 2 |
| 2. | PMA332 | Finite Element Methods | 3 | 0 | 2 |
| 3. | PMA333 | Introduction to Astronomy and Astrophysics | 3 | 2 | 0 |
| 4. | PMA334 | Wavelet and Applications | 3 | 0 | 2 |
| 5. | PMA335 | Mathematical Biology and Non-Linear Dynamics | 3 | 2 | 0 |
| 6. | PMA336 | Operator Theory | 3 | 2 | 0 |
| 7. | PMA337 | Enumerative Combinatorics | 3 | 2 | 0 |
| 8. | PMA338 | Fuzzy Sets and Applications | 3 | 2 | 0 |
| 9. | PMA339 | Fluid Mechanics | 3 | 2 | 0 |
| 10. | PMA340 | Discrete Mathematical Structures | 3 | 2 | 0 |

## Elective II [4 Credits]

| S.No. | Course No. | Course Name | L | T | P |
|---|---|---|---|---|---|
| 1. | PMA432 | Modelling of Stellar Structure | 3 | 2 | 0 |
| 2. | PMA433 | Asymptotic Methods and Perturbation Theory | 3 | 2 | 0 |
| 3. | PMA434 | Theory of Elasticity | 3 | 2 | 0 |
| 4. | PMA435 | Algebraic Coding Theory | 3 | 2 | 0 |
| 5. | PMA436 | Topological Vector Space | 3 | 2 | 0 |
| 6. | PMA437 | Fixed Point Theory | 3 | 2 | 0 |
| 7. | PMA438 | Statistical Simulation and Computation | 3 | 0 | 2 |
| 8. | PMA439 | Financial Mathematics | 3 | 0 | 2 |
| 9. | PMA440 | Combinatorial Programming | 3 | 2 | 0 |
| 10. | PMA441 | Stochastic Calculus | 3 | 2 | 0 |
| 11. | PMA442 | Advanced Numerical Optimization Techniques | 3 | 2 | 0 |

## PMA107 Real Analysis

**Credits: 3.5 [3-1-0]**

**Course Objectives:** The aim of this course is to introduce the students real number system and metric spaces. In particular, the notion of completeness, compactness, limit, continuity, differentiability, integrability and uniform continuity.

**Contents:**

**Real Number System and Set Theory:** Completeness property, Archimedean property, Denseness of rationals and irrationals, Countable and uncountable sets, Cardinality.

**Metric Spaces:** Open and closed sets, Interior, Closure and limit points of a set, Subspaces, Continuous functions on metric spaces, Convergence in a metric space, Complete metric spaces, Compact metric spaces, Compactness and uniform continuity, Connected metric spaces, Total boundedness, Finite intersection property.

**Sequence and Series of Functions:** Pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weierstrass $M$-test, Abel's and Dirichlet's tests for uniform convergence, Uniform convergence and continuity, Uniform convergence and differentiation, Weierstrass approximation theorem.

**Riemann-Stieltjes Integral:** Definition and existence of Riemann-Stieltjes integral, Properties, Integration and differentiation, Fundamental theorem of calculus.

**Course Learning Outcomes:** The student will be able to

1. analyze different properties of $\mathbb{R}$.
2. apply properties viz. convergence, completeness, compactness etc. from the real line to metric spaces.
3. identify the difference between pointwise and uniform convergence and analyze the effect of uniform convergence on the functions with respect to continuity, differentiability and integrability.
4. determine the Riemann-Stieltjes integrability of a bounded functions.

**Text/References:**


- W. Rudin, Principles of Mathematical Analysis, McGraw-Hill, 3rd edition, 2013.
- G. F. Simmons, Introduction to Topology and Modern Analysis, Tata McGraw Hill, 2008.
- S. C. Malik and S. Arora, Mathematical Analysis, Wiley Eastern, 2010.
- P. K. Jain and K. Ahmad, Metric Spaces, Alpha Science Publishers, 2004.
- R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 4th ed., 2010.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes etc.) | 25% |

## PMA108 Algebra I

**Credits: 3.5 [3-1-0]**

**Course Objectives:** The course intends to impart knowledge in the areas of Group Theory and Linear Algebra with an aim of enabling students to apply the concepts learned to other areas.

**Contents:**

**Linear Transformations:** Review of vector spaces, Linear transformations, Algebra of linear transformations, Matrix representation of linear transformations, Different types of matrices, Change of basis, Rank, trace and determinant of a matrix, Dual and double dual, Transpose of a linear transformation, Linear transformations and their characteristic roots and vectors, Algebraic and geometric multiplicity, Cayley-Hamilton theorem, Minimal polynomial, Canonical forms, Diagonal form, Triangular form, Rational and Jordan form.

**Group Theory:** Center, Normalizer, Centralizer, Homomorphism and isomorphism, Cyclic groups and their Properties, Classification of subgroups of cyclic groups, Permutation groups, Cyclic notation of permutation groups, Cayley's theorem, Conjugate elements, Class equation, Direct product of groups, Fundamental theorem of finite abelian groups, Cauchy's theorem, Sylow theorems and its applications.

**Course Learning Outcomes:** The students will be able to

1. find characteristic roots, vectors, and quadratic forms of matrices.
2. recognize various canonical forms of linear transformations.
3. describe the properties of permutation groups and cyclic groups.
4. apply structure theory of abelian groups and Sylow theorems to solve different problems.

**Text/References:**


- K. Hoffmann and R. Kunze, Linear Algebra, Pearson, Second edition, 2014.
- I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd., 2005.
- S. Singh and Q. Zameeruddin, Modern Algebra, Vikas Publishing House, 2006.
- S. Luthar and I. B. S. Passi, Algebra (Vol. 1 and 2), Narosa Publishing House, 1999.
- J. A. Gallian, Contemporary Abstract Algebra, Cengage Learning, 8th ed., 2013.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes etc.) | 25% |

## PMA109 Ordinary Differential Equations

**Credits: 3.5 [3-1-0]**

**Course Objectives:** The main objective is to provide mathematics majors with an introduction to the theory of Ordinary Differential Equations (ODEs) through applications and methods of solution. Students will become knowledgeable about system of ODEs and how they can serve as models for physical processes. The course will also develop an understanding of the elements of analysis of ODEs.

**Contents:**

**Qualitative Properties of Solutions:** Existence and uniqueness of initial value problems for first order equations, Picard's existence theorem, non-local existence of solutions, Gronwall's inequality and uniqueness, continuation of solutions and continuous dependence, Peano's existence theorem.

**Sturm Theory:** Wronskian and fundamental solution, Sturm separation theorem and oscillations, Sturm comparison theorem and applications.

**Boundary Value Problems:** Two-point boundary value problems, Sturm-Liouville theory, Lagrange's identity, eigenvalues and eigenfunctions of the regular and periodic Sturm-Liouville problems, eigenfunction expansion, non-homogeneous problems, singular Sturm-Liouville problems, Green's function.

**Linear Systems:** Origin of system, linear system with constant coefficients, existence of solutions, Wronskian, fundamental solutions, Abel's formula, fundamental matrices, matrix exponential form of solution, variation of parameters, linear system with variable coefficients.

**Nonlinear Systems and Stability:** Phase plan, autonomous system, critical points and stability, stability by Liapunov's direct method, simple critical points, periodic solutions: Poincare-Bendixson theorem, limit cycles.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. give examples of differential equations for which either existence or uniqueness of solution fails.
2. state correctly and apply basic facts of Sturm separation and comparison theorems.
3. solve boundary value problems and Sturm-Liouville problems.
4. state correctly and apply basic facts of systems: eigen-values and eigen-vector, fundamental matrices, and non-homogeneous systems.
5. sketch the phase portraits and apply standard methods to check the stability of critical points for autonomous system.

**Text/References:**


- William E. Boyce, Richard C. DiPrima, and Douglas B. Meade, Elementary Differential Equations and Boundary Value Problems, 11th edition, Wiley, 2017.
- E. A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, McGraw Hill Education, 2017.
- George F. Simmons and Steven G. Krantz, Differential Equations: Theory, Technique, and Practice, McGraw Hill Education, 2006.
- L. Perko, Differential Equations and Dynamical Systems, 3rd Edition, Springer, 2008.

**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA110 Mechanics

**Credits: 3.5 [3-1-0]**

**Course Objectives:** This course is intended to provide a treatment of basic knowledge in mechanics used in deriving a range of important results and problems related to rigid bodies The objective is to provide the student the classical mechanics approach to solve a mechanical problem.

**Contents:**

**Dynamics of a Particle:** Tangential and normal accelerations, Simple harmonic motion, Oscillatory motion projectile motion, Central forces, Apses and apsidal distances, Stability of orbits, Kepler's laws of planetary motion, Simple pendulum, Motion in a resisting medium, Motion of a pendulum in a resisting medium.

**Linear and Angular Momentum:** Rate of change of angular momentum for a system of particles, Impulsive forces, Moments and products of inertia of a rigid body, Equimomental system, Principal axes, Coplanar distribution, General equations of motion.

**Motion About a Fixed Axis:** Compound pendulum, Motion in two dimensions, Euler's dynamical equations and simple stability considerations.

**Lagrangian and Hamiltonian Mechanics:** Constrained motion, D'Alemberts principle, Variational Principle, Lagrange's equations of motion, Generalized coordinates, cyclic coordinates, Hamiltons principles, Principles of least action, Hamilton's equation of motion, Phase Space, State space examples, Cannonical transformations, Lagrange's and Poisson brackets invariance.

**Course Learning Outcomes:** The student will be able to

1. describe the dynamics involving a single particle like projectile motion, Simple harmonic motion, pendulum motion and related problems.
2. analyze the path described by the particle moving under the influence of central force.
3. apply the concept of system of particles in finding moment inertia, directions of principle axes and consequently Eulers dynamical equations for studying rigid body motions.
4. obtain the equation of motion for mechanical systems using the Lagrangian and Hamiltonian formulations of classical mechanics.
5. obtain canonical equations using different combinations of generating functions.

**Text/References:**


- F. Chorlton, Text book of Dynamics, CBS Publishers, 1985.
- J. L. Synge and B. A. Griffith, Principles of Mechanics, Tata McGraw Hill, 1971.
- C. Fox, An Introduction to the Calculus of Variations, Dover Publications, 1992.
- H. Goldstein, C. Poole, and J. Safko, Classical Mechanics, Addison Wesley, 2002.
- P. Mann, Lagrangian and Hamiltonian Dynamics, Oxford University Press, 2018.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes etc.) | 25% |

## PMA111 Computer Programming

**Credits: 4 [3-0-2]**

**Course Objectives:** This course is designed to explore computing and to show students the art of computer programming. Students will learn some of the design principles for writing good programs.

**Course Outlines:**

**Computer Fundamentals:** Introduction to computer systems, number system, integer, signed integer, fixed and floating point representations, IEEE standards, integer and floating point arithmetic; CPU organization, ALU, registers, memory, the idea of program execution at micro level.

**Algorithms and Programming Languages:** Algorithm, flowcharts, pseudocode, generation of programming languages.

*C* **Language:** Structure of *C* Program, life cycle of program from source code to executable, compiling and executing C code, keywords, identifiers, primitive data types in *C*, variables, constants, input/output statements in C, operators, type conversion and type casting, conditional branching statements, iterative statements, nested loops, break and continue statements.

**Functions:** Declaration, definition, call and return, call by value, call by reference, showcase stack usage with help of debugger, scope of variables, storage classes, recursive functions, recursion vs Iteration.

**Arrays, Strings and Pointers:** One-dimensional, two-dimensional and multi-dimensional arrays, operations on array: traversal, insertion, deletion, merging and searching, Inter-function communication via arrays: passing a row, passing the entire array, matrices. Reading, writing and manipulating Strings, Understanding computer memory, accessing via pointers, pointers to arrays, dynamic allocation, drawback of pointers.

**Object Oriented Programming Concepts:** Data hiding, abstract data types, classes, access control; class implementation, constructors, destructor operator overloading, friend functions; object oriented design (an alternative to functional decomposition) inheritance and composition; dynamic binding and virtual functions; polymorphism; dynamic data in classes.

**Laboratory work:** To implement Programs for various kinds of programming constructs in *C* Language.

**Course Learning Outcomes:** On successful completion of this module, students will be able to:

1. comprehend concepts related to computer hardware and software, draw flowcharts and write algorithm/pseudocode.
2. write, compile and debugprograms in *C* language, use different data types, operators and console I/O function in a computer program.
3. design programs involving decision control statements, loop control statements, case control structures, arrays, strings, pointers, functions and implement the dynamics of memory by the use of pointers.
4. comprehend the key concepts of object-oriented design and programming concepts.

**Text/References:**


- Y. Kanetkar, Let Us C, BPB Publications, 2nd ed., 2016.
- Brian W. Kernighan, Dennis M. Ritchie, The C Programming Language, Pearson, 2nd ed, 2015.
- H. M. Deitel and P. J. Deitel, C++ How to Program, Prentice Hall, 8th Ed, 2011.
- E. Balaguruswamy, Object Oriented Programming with C++, McGraw Hill, 2013.

**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 25% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes/Lab Evaluation) | 30% |

## PMA204 Measure Theory and Integration

**Credits: 3.5 [3-1-0]**

**Course Objectives:** The aim of the course is to introduce the students Lebesgue measure, measurable sets and their properties, measurable functions and their properties and Lebesgue integral.

**Course Outlines:**

**Lebesgue Measure:** Introduction, Outer measure, Lebesgue measure, measurable sets, Properties of measurable sets, Borel sets and their measurability, non-measurable sets.

**Measurable Functions:** Definition and properties of measurable functions, step functions, characteristic functions, simple functions, Littlewood's three principles, convergence in measure.

**Lebesgue Integral:** Lebesgue integral of bounded function, Integration of non-negative functions, General Lebesgue integrals, Integration of series, Comparison of Riemann and Lebesgue integrals.

**Differentiation and Integration:** Differentiation of monotone functions, functions of bounded variation, Lebesgue differentiation theorem, differentiation of an integral, absolute Continuity.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. define Lebesgue measure on $\mathbb{R}$.
2. describe measurable functions and its properties.
3. apply measures to construct integrals and explain convergence theorems for the Lebesgue integral.
4. analyze the relation between differentiation and Lebesgue integration.

**Text/References:**


- H. L. Royden and P. M. Fitzpatrick, Real Analysis, Pearson Education, 4th Edition, 2011.
- G. de Barra, Measure Theory and Integration, Wiley Eastern Ltd., 2012.
- P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, New Age International Ltd., 2nd Edition, 2010.
- I. K. Rana, An Introduction to Measure and Integration, Narosa Publication House, 2nd Edition, 2010.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA205 Algebra II

**Credits: 3.5 [3-1-0]**

**Course Objectives:** The course intends to impart knowledge in the areas of ring theory and field theory with an aim of enabling students to apply the concepts learned to other areas.

**Course Outlines:**

**Ring Theory:** Special kinds of rings, Subrings and ideals, Homomorphism and isomorphism, Quotient rings, Prime and maximal ideals, Characteristic of a ring, Integral domain, Units and zero divisors, Polynomial rings, Irreducible and prime elements, Unique factorization domain, Principal ideal domain, Euclidean domain.

**Field Theory:** Polynomials and their irreducibility criteria, Adjunction of roots, Finite and infinite extensions, Algebraic and transcendental extensions, Algebraically closed fields, Splitting fields, Normal extension, Multiple roots, Finite fields, Separable and inseparable extensions, Automorphism groups and fixed fields, Fundamental theorem of Galois theory.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. describe the properties of integral domain, principle ideal domain, Euclidean domain and unique factorization domain.
2. apply different irreducibility criteria to check the irreducibility of a polynomial.
3. describe the concepts of fields, various extensions of fields and splitting fields.
4. analyze the properties of finite fields and Galois theory.

**Text/References:**


- I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd., 2005
- S. Singh and Q. Zameeruddin, Modern Algebra, Vikas Publishing House, 2006.
- J. A. Gallian, Contemporary Abstract Algebra, Cengage Learning, 8th ed, 2013.
- P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul, Basic Abstract Algebra, Cambridge University Press, 1997.
- I. S. Luthar and I. B. S. Passi, Algebra (Vol. 4), Narosa Publishing House, 2004.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

**Credits: 3.5 [3-1-0]**

**Course Objectives:** Partial Differential Equations (PDEs) are at the heart of applied mathematics and many other scientific disciplines. The primary goal of this course is to provide students the knowledge about fundamental concepts of PDE theory and analytical methods for solving PDEs. Students will become knowledgeable about PDEs and how they can serve as models for physical processes such as Laplace equation, wave equation and transport phenomena including diffusion. The course will also develop an understanding of the elements of analysis of PDEs.

**Course Outlines:**

**First-Order PDEs:** Origin of first-order equations, linear and quasi-linear equations, Cauchy problem, characteristics, general solution, integral surfaces, first-order nonlinear equations, Cauchy's method of characteristic, complete integrals, singular solutions, separation of variables, Cauchy-Kowalewsky theorem.

**Second-Order PDEs:** Linear equations with constant coefficients, classification, characteristic and canonical forms, equations with variable coefficients.

**Wave Equation:** Derivation of wave equation, solution of wave equation, initial value problem and d'Alambert's principle, causality, energy and uniqueness, non-homogeneous waves and Duhamel's principle, boundary value problems.

**Heat Equation:** Solution of heat equation, fundamental solution and its properties, Cauchy problem, maximum principle, energy methods, uniqueness, boundary value problems for heat with Dirichlet, Neumann, and mixed boundary conditions.

**Laplace Equation:** Fundamental solution, boundary value problems, maximum principles, Poisson's formula, properties of harmonic functions, separation of variables in polar form: wedges, annuli, and exterior of a circle.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. state correctly and apply to examples the basic facts about the first order PDEs and method of characteristics.
2. formulate, classify, and transform second-order linear partial differential equations into canonical form.
3. state correctly and apply to examples the basic facts about the Wave equation: d'Alambert's principle, energy method, and Duhamel's principle.
4. study heat equation and can apply maximum principle, energy methods, and able to solve boundary value problems.
5. state correctly and apply to examples the basic facts about the Laplace equation: maximum principle, Poisson's formula, boundary value problems.

**Text/References:**


- Walter A. Strauss, Partial Differential Equations: An Introduction, Willey, 2nd Edition, 2007.
- L. C. Evans, Partial Differential Equations, American Mathematical Society, 2nd Edition, 2010.
- I. N. Sneddon, Elements of Partial Differential Equations, Dover Publications, 2006.

- Tyn Myint-U and L. Debnath, Linear Partial Differential Equations for Scientists and Engineers, Birkhauser, 4th edition, 2007.
- F. John, Partial Differential Equations, 4th edition, Springer, 1991.

**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA207 Complex Analysis

**Credits: 3.5 [3-1-0]**

**Course Objectives:** The course aims to introduce the theory of complex analysis to graduate students with applications to solve problems in the mathematical sciences and engineering.

**Course Outlines:**

**Complex Numbers:** Introduction to complex numbers, geometrical interpretation, different representations of complex numbers, Stereographic projection.

**Elementary and Analytic Functions:** functions of complex variables, examples of elementary functions like exponential, trigonometric and hyperbolic functions, elementary calculus on the complex plane (limits, continuity, differentiability), Cauchy-Riemann equations, analytic functions, harmonic functions with examples, branch points and branch cuts, multi-valued functions (eg. logarithmic function and its branches, Riemann surfaces).

**Complex Integration:** Cauchy's integral theorem, Cauchy integral formula for higher derivatives, Morera's theorem, Liouville's theorem, maximum-modulus principle, Schwarz lemma.

**Series Expansion of Complex Functions:** Power series, Taylor and Laurent series of complex functions, convergence, definition of holomorphic and meromorphic functions, zeros and poles, classification of singular points, removable singularities, Weierstrass theorems ($M$ test and factor theorem), argument principle and Roche's theorem (eg. with application to prove the fundamental theorem of algebra).

**Residue Calculus:** General form of Cauchy's theorem, Cauchy residue theorem, evaluation of definite integrals using residue theorem (principal value integrals and integrals with branch points), residue at infinity.

**Conformal Mappings:** Elementary conformal mappings (Schwarz-Christoffel transformation), analytic continuation, method of analytic continuation by power series (e.g. application in defining the Riemann-Zeta function).

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. represent complex numbers in Cartesian, polar and matrix form, geometrical interpretation of complex numbers.
2. inspect the analyticity of complex functions including the utility of Cauchy-Riemann equations, evaluation of contour integrals using Cauchy integral formula.
3. represent complex functions as power series (e.g., Taylor and Laurent) and their convergence, classification of singularities.
4. apply residue calculus using Cauchys residue theorem and method of analytic continuation.
5. have knowledge of conformal maps.

**Text/References:**


- M. Ablowitz and A. S. Fokas, Complex Variables: Introduction and Applications, Cambridge University Press, 2nd edition, 2003.
- R. V. Churchill and J. W. Brown, Complex Variable and Applications, McGraw Hill, 8th edition, 2009.
- L. V. Ahlfors, Complex Analysis, Tata McGraw Hill, 3rd edition, 1979.

- H. S. Kasana, Complex Variables: Theory and Applications, Prentice Hall India, 2nd edition, 2005.
- S. Ponnuswamy, Foundation of Complex Analysis, Narosa Publishing House, 2nd edition, 2011.

**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA208 Numerical Analysis

**Credits: 4.5 [3-1-2]**

**Course Objectives:** The primary goal is to provide mathematics majors with a basic knowledge of numerical methods including: root finding, numerical linear algebra, interpolation, integration, solving systems of linear equations, curve fitting, and numerical solution to ordinary differential equations. MATLAB is the software environment used for implementation and application of these numerical methods. The course will also develop an understanding of the elements of error analysis for numerical methods and certain proofs. The course will further develop problem solving skills.

**Course Outlines:**

**Mathematical Preliminaries and Error Analysis:** Round off errors, Algorithms and convergence, conditioning and stability.

**Numerical Solution of Nonlinear Equations:** Review of bisection, fixed point and secant method, Newton's method and its variants, convergence analysis, zeros of polynomials and Muller's method, Newton's method in higher dimensions.

**Solutions of Linear Systems:** Direct methods, Gauss-elimination method, pivoting, matrix factorization. Iterative methods: Matrix norms, Jacobi and Gauss-Seidel, Relaxation methods and their convergence, error bounds and iterative refinement. Computation of eigenvalues and eigenvectors: power method, Householder's method, QR algorithm.

**Interpolation:** Lagrange interpolation, Newton's divided differences, Hermite interpolation, Splines, Richardson's extrapolation.

**Numerical Integration:** Newton-Cotes formula, Trapezoidal and Simpson's rules, Gaussian quadrature, Romberg integration, Multiple integrals.

**Numerical Solution of Ordinary Differential Equations:** Initial value problems, Euler's method, Higher-order methods of Runge-Kutta type, Multi-step methods, Adams-Bashforth, Adams-Moulton and Milne's methods, convergence and stability analysis, system of equations. Boundary value problems: Shooting methods, finite differences, Rayleigh-Ritz methods.

**Lab Experiment:** Implementation of numerical techniques using MATLAB based on course contents.

**Course Learning Outcomes:** On successful completion of this module, students will be able to:

1. find the source of errors and its effect on any numerical computations and be familiar with finite precision computations.
2. solve an algebraic or transcendental equation using an appropriate numerical method and perform an error analysis for a given numerical method.
3. solve a linear system of equations using an appropriate numerical method which include direct and iterative methods and apply numerical methods to find eigen-values and corresponding eigen-vectors.
4. approximate the given data with an appropriate interpolating polynomial.
5. calculate a definite integral numerically and solve initial and boundary value problems using appropriate numerical methods.

**Text/References:**


- Richard L. Burden, J. Douglas Faires and Annette Burden, Numerical Analysis, Cengage Learning, 10th edition, 2015.

- E. Ward Cheney and David R. Kincaid, Numerical Mathematics and Computing, Cengage Learning, 7th edition, 2012.
- K. Atkinson and W. Han, Elementary Numerical Analysis, John Willey and Sons, 3rd edition, 2004.
- Endre Suli and David F. Mayers, An Introduction to Numerical Analysis, Cambridge University Press, 2003.

**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 25% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes/Lab Evaluation) | 30% |

## Third Semester

## PMA301 Functional Analysis

**Credits: 3.5 [3-1-0]**

**Course Objectives:** Functional analysis is a fundamental area of pure mathematics, with countless applications to the theory of differential equations, engineering, and physics. The students will be exposed to the theory of Banach space, Hilbert spaces, linear transformations and functionals. In particular, the major theorems in functional analysis, namely, Hahn-Banach theorem, uniform boundedness theorem, open mapping theorem and closed graph theorem will be covered.

**Course Outlines:**

Review of some basic concepts in metric spaces and topological spaces, completeness proofs and completion of metric spaces.

Normed linear spaces and Banach spaces, examples of Banach spaces, quotient spaces, equivalent norms, finite dimensional Banach spaces and compactness, bounded and continuous linear operators and linear functionals, dual space, Banach fixed-point theorem and applications, Hahn-Banach theorem and applications, uniform boundedness theorem, open mapping and closed graph theorem.

Inner product spaces and its properties, Hilbert spaces and examples, best approximation in Hilbert spaces, orthogonal complements, orthonormal basis, Gram Schmidt orthogonalisation, dual of a Hilbert space.

Operator theory, adjoint of an operator, Riesz representation theorem, self-adjoint operators, normal and unitary operators, projection operator.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. recognize the fundamentals of normed linear spaces and completeness property.
2. understand the fundamentals of Hilbert spaces and can provide best approximations in Hilbert spaces.
3. represent the functionals/dual of a normed linear space and inner product space.
4. define and thoroughly explain self-adjoint, normal and unitary operators and analyze operators from applications.

**Text/References:**


- E. Kreyszig, Introductory Functional Analysis with Applications, Wiley, 2007.
- John B. Conway, A Course in Functional Analysis, Springer, 2nd edition, 1990.
- G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill Education, 2017.
- S. Kesavan, Functional Analysis, Hindustan Book Agency, 2014.
- Peter D. Lax, Functional Analysis, John Wiley & Sons, 2002.
- W. Rudin, Functional Analysis, McGraw Hill, 1991.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA302 Topology

**Credits: 3.5 [3-1-0]**

**Course Objectives:** To introduce the fundamentals Point Set Topology which is needed in several areas of Mathematics.

**Course Outlines:**

**Topological Spaces:** Review of Metric spaces, Definition and Examples of topological spaces, Topology induced by a metric, open and closed sets, Closure, Neighbourhood, Limit point, Derived set, Interior, Exterior and Boundary points. Bases, Sub-bases and examples, topology generated by sub-bases, Subspaces and relative topology; continuous function and Homeomorphism.

**Countability and Separation Axioms:** First and second countable spaces, Separable Spaces, Lindeloff spaces. Separation axioms ($T_0$, $T_1$, Hausdorff spaces, regular and Normal spaces), Urysohn's Lemma, Metrization Theorem, Tietze extension theorem.

**Compactness and Connectedness:** Compact spaces and their basic properties; Connected spaces, Connected sets in the real line, Intermediate value theorem, Connected components, Path connected components, Locally connected spaces, Totally disconnected spaces; Continuous functions and connected sets.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. define various topologies on a general set.
2. use the concepts of bases, subbases to prove the results and theorems.
3. know about the various topological spaces, i.e. countable, separation axioms.
4. derive results related to compactness and connectedness.

**Text/References:**


- J. R. Munkres, Topology: A First Course, Prentice-Hall, 2007.
- J. L. Kelley, General Topology, Springer, 1955.
- G. F. Simmons, Introduction to Topology and Modern Analysis, Tata McGraw Hill, 1963.
- K. D. Joshi, Introduction to General Topology, Wiley, 1983.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA303 Probability and Statistics

**Credits: 4.5 [3-1-2]**

**Course Objectives:** The course aims to shape the attitudes of learners regarding the field of statistics. Specifically, the course aims to i) motivate students towards an intrinsic interest in statistical thinking, ii) instill the belief that statistics is important for scientific research.

**Course Outlines:**

**Introduction & Random variables:** Classical and Empirical Probability, Axioms of probability, probability space, conditional probability, independence, Baye's rule, Random variable, some common discrete and continuous distributions (Binomial, Poisson, Geometric, Negative binomial, Rectangular, Exponential, Normal, Gamma).

**Mathematical Expectation:** Moments, Moment generating functions, Characteristic function and properties.

**Bivariate Probability Distribution:** Probability distribution of functions of a random variable, joint and marginal distributions, conditional distributions.

**Limit Theorems:** Modes of convergence, Markov and Chebyshev's inequalities, Law of large numbers, Central limit theorem.

**Correlation and Regression:** Covariance, Karl-Pearson and rank Correlation coefficients; linear regression between two variables.

**Hypothesis tests:** Introduction to Sampling Distribution (standard normal, chi-square, $t$ & $F$ distributions), Theory of Estimation, Properties of an estimator, Tests for Goodness of fit, Method of maximum likelihood, Method of moments, Critical regions, Type I and II errors, Neyman-Pearson lemma (without proof).

**Parametric & Non-parametric tests:** $T$-test, $Z$-test, Chi-square test, Sign Test, Wilcoxon Signed-rank Test, Kolmogorov Smirnov Test.

**Laboratory Work:** Lab work will be based on the programming in MATLAB/SPSS/R language of various statistical techniques.

**Minor Projects:** The minor projects will be set in consonance with material covered in theory and laboratory classes.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. compute probabilities of events along with an understanding of random variables and distribution functions and expectation.
2. understand the convergence of sequence in probabilities
3. analyze the correlated data and fit the linear regression models.
4. make statistical inferences using principles of hypothesis tests.

**Text/References:**


- P. L. Meyer, Introduction to Probability and Statistical Applications, Oxford & IBH, 2007.
- A. M. Goon, M. K. Gupta, and B. Dasgupta, An Outline of Statistical Theory, Vol. I, The World Press Pvt. Ltd., 2000.

- R. V. Hogg, and A. T. Craig, Introduction to Mathematical Statistics, Prentice Hall of India, 2004.
- R. E. Walpole, R. H. Myers, S. L. Myers, and K. Ye, Probability and Statistics for Engineers and Scientists, Pearson, 2010.
- R. A. Jhonson, C. B. Gupta, Miller and Freund's Probability and Statistics for Engineers, Dorling Kindersley, 7 ed., 2007


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 25% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes/Lab Evaluation) | 30% |

## PMA304 Mathematical Programming

**Credits: 4.5 [3-1-2]**

**Course Objectives:** Operations research helps in solving problems in different environments that needs decisions. This module aims to introduce students to use quantitative methods and techniques for effective decisionsmaking; model formulation and applications that are used in solving business decision problems.

**Course Outlines:**

**Linear programming:** Linear programs formulation through examples from engineering / business decision making problems, preliminary theory and geometry of linear programs, basic feasible solution, simplex method, variants of simplex method, like two phase method and revised simplex method; duality and its principles, interpretation of dual variables, dual simplex method, primal-dual method.

**Integer programming problems:** Linear integer programs, their applications in real decision making problems, cutting plane and branch and bound methods.

**Transportation and Assignment problem:** Initial basic feasible solutions of balanced and unbalanced assignment/transportation problems, optimal solutions, time minimization assignment/transportation problem. Game Theory: Two person zero-sum game, Game with mixed strategies, Dominance property, solution by linear programming.

**Nonlinear Programming:** Concepts of convexity and its generalizations, Maxima and minima of convex functions, unconstrained optimization problems, constrained programming problems, Lagrange's multiplier rule and Kuhn-Tucker conditions for constrained programming problems. quadratic programs, Wolfe method, applications of quadratic programs in some domains like portfolio optimization and support vector machines, etc.

**Laboratory Work:** Laboratory experiments will be set in consonance with the materials covered in theory.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. formulate and solve some real life problems as Linear programming problem.
2. use the simplex method to find an optimal vector for the standard linear programming problem and the corresponding dual problem.
3. find optimal solution of transportation problem and assignment problem
4. solve two person zero sum game.
5. solve unconstrained and constrained nonlinear programming problems.

**Text/References:**


- S. Chandra, Jayadeva, and A. Mehra, Numerical Optimization and Applications, Narosa Publishing House, 2013.
- R. W. Cottle and M. N. Thapa, Linear and Nonlinear Optimization, Springer, 2017.
- M. S. Bazaraa, J. J. Jarvis, and H. D. Sherali, Linear Programming and Network Flows, John Wiley and Sons, 1990.
- M. S. Bazaraa, H. D. Sherali, and C. M. Shetty, Nonlinear Programming: Theory and Algorithms, John Wiley and Sons, 1993.

- M. Ferris, O. Mangasarian, and S. Wright, Linear Programming with Matlab, MPS-SIAM series on Optimization, 2007.
- G. Hadley, Linear Programming, Narosa Publishing House, 2002.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 25% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes/Lab Evaluation) | 30% |

## Fourth Semester

## PMA401 Number Theory

**Credits: 3.5 [3-1-0]**

**Course Objectives:** The purpose of this course is to introduce classical number theory, primes, congruences, solution of congruences, arithmetic functions, quadratic residues and primitive roots. Apart from teaching the theory, stress will be on solving problems and helping students to understand applications of number theory in cryptography and in other fields.

**Course Outlines:**

**Divisibility and Congruences:** Least and greatest common divisor, Fundamental theorem of arithmetic, congruence, residue classes, Chinese remainder theorem, congruences with prime moduli, Fermat's theorem, Euler's theorem and Wilson Theorem, applications to cryptography.

**Arithmetic Functions:** $\phi(x)$, $d(x)$, $\mu(x)$, $\sigma(x)$, Mobius inversion formula and greatest integer function, the symbol small "oh", big "oh" and their basic properties. Perfect numbers, Mersenne primes and Fermat numbers.

**Polynomial Congruences:** Primitive roots, indices and their applications, Quadratic residues, Legendre symbol, Euler's criterion, Gauss's Lemma, Quadratic reciprocity law, Jacobi symbol, Binary quadratic forms and their reduction, sums of two and four squares, positive definite binary quadratic forms.

**Diophantine Equations:** Sums of two and four squares, Pell's equations and Fermat's Last Theorem (statement only).

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. apply the properties of divisibility and prime numbers to prove results.
2. solve the linear congruences and apply in cryptography.
3. prove results using the properties of arithmetic functions.
4. implement the theory to solve polynomial congruences.
5. analyze the integral solutions of system using Diophantine equations and sum of squares.

**Text/References:**


- I. Niven, S. H. Zuckerman, and L. H. Montgomary, An Introduction to Theory of Numbers, John Wiley and Sons, 1991.
- H. Davenport, Higher Arithmetic, Cambridge University Press, 1999.
- David M. Burton, Elementary Number Theory, Wm. C. Brown Publishers, Dubuque, Ivova, 1989.
- T. M. Apostol, Introduction to Analytic Number Theory, Springer-Verlag, 1998.
- G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers, Oxford University Press, 1979.
- J. B. Dence and T. P. Dence, Elements of the Theory of Numbers, Academic Press, 1999.
- Johannes A. Buchmann, Introduction to Cryptography, Springer Verlag, 2001.

**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes etc.) | 25% |

## PMA402 Mathematical Methods

**Credits: 3.5 [3-1-0]**

**Course Objectives:** This course isintendedto prepare the studentwithmathematicaltools and techniques that are required in advanced coursesoffered in the applied mathematics and engineering programs. The objective of this course is to enable students to apply transforms and variation problem technique for solving differential equations and extremum problems.

**Course Outlines:**

**Laplace Transform:** Review of Laplace transform, Applications of Laplace transform in initial and boundary value problems, Heat equation, Wave equation, Laplace equation.

**Fourier Series and Transforms:** Definition, Properties, Solutions of differential equations using Fourier series. Fourier integral theorem, Convolution theorem and Inversion theorem, Discrete fourier transforms (DFT), Relationship of FT and fast Fourier transforms (FFT), Linearity, Symmetry, Time and frequency shifting, Convolution and correlation of DFT. Applications of FT to heat conduction, Vibrations and potential problems, Z-transform, Parseval's theorems.

**Integral Equations:** Linear integral equations of the first and second kind of fredholm and volterra type, Conversion of linear ordinary differential equations into integral equations, Solutions by successive substitution and successive approximation, Neumann series and resolvent kernel methods.

**Calculus of Variation:** The extrema of functionals, The variation of a functional and its properties, Euler equations in one and several independent variables, Field of extremals, Sufficient conditions for the extremum of a functional conditional extremum, Moving boundary value problems, Initial value problems, Ritz method.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. solve initial and boundary value problems using Laplace transformation.
2. apply Fourier transformation and Z transformation to the relevant problems.
3. solve initial and boundary value problems using Fourier series.
4. find solutions of linear integral equations of first and second type (Volterra and Fredhlom).
5. obtain solution of initial and boundary value problems using theory of calculus of variations.

**Text/References:**


- G. F. Simmons, Differential Equations with Applications and Historical Notes, Tata McGraw Hill, 1991.
- I. M. Gelfand and S. V. Fomin, Calculus of Variations, Prentice Hall, 1963.
- Ram P. Kenwal, Linear Integral Equations: Theory and Techniques, Academic Press, 1971.
- I. N. Sneddon, The Use of Integral Transforms, Tata McGraw Hill, 1985.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## Elective I

## PMA331 Numerical Methods for Partial Differential Equations

**Credits: 4 [3-0-2]**

**Course Objectives:** This course deals with the mathematical theory of numerical methods especially finite difference scheme used to solve second partial differential equations (PDEs) arising in many real world applications. The course covers not only the algorithms and numerical schemes but also deals with the error analysis associated with them. The main aim is to give the students the basic understanding of numerical treatment of partial differential equations.

**Course Outlines:**

**Hyperbolic Equations:** Hyperbolic equations in one and two dimension, advection equation, wave equation, hyperbolic conservation laws, Burgers equation, KdV equation, CFL conditions, Upwind and Lax Wendroff finite difference approximations.

**Parabolic Equations:** Finite difference schemes for parabolic equations of second order in one space variable with constant coefficients two and three levels explicit and implicit difference schemes, Truncation errors and convergence analysis, second order parabolic equations in two space variable with constant coefficients-improved explicit schemes, Implicit methods, alternating direction implicit (ADI) methods, Difference schemes for parabolic equations with variable coefficients in one and two space dimensions.

**Elliptic Equations:** Numerical solutions of elliptic equations, Approximations of Laplace operators, Solutions of Laplace and Poisson equations with Dirichlet, Neumann and mixed boundary in rectangular, circular and triangular regions.

**Course Learning Outcomes:** On successful completion of this module, students will be able to:

1. enumerate the numerical solution of hyperbolic equations of second order in one and two space variables with explicit and implicit methods.
2. find numerical solutions of heat conduction diffusion equation in one and two dimension.
3. carry out the stability analysis and truncation error in various aforementioned numerical schemes.
4. solve the Dirichlet, Neumann and mixed type problems with Laplace and Poisson equations in different regions like rectangular, circular and triangular.

**Text/References:**


- K. W. Morton and D. F. Mayers, Numerical solution of Partial Differential Equations, 2nd Ed. Cambridge University Press, 2005.
- John C. Strikewerda, Finite Difference Schemes and Partial Differential Equations, 2nd Ed. SIAM, 2004.
- H. P. Langtangen, Computational Partial Differential Equations, Springer Verlag, 2003.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 25% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes/Lab Evaluation) | 30% |

**Credits: 4 [3-0-2]**

### Course Objectives:

The objective of the course includes an introduction about different finite element methods in one-, two- and three-dimensions. The course focuses on analyzing variety of finite elements as per the requirements of solutions of differential equations.

### Course Outlines:

**Introduction:** Finite element methods, History and range of applications.

**Finite Elements:** Definition and properties, Assembly rules and general assembly procedure, Features of assembled matrix, Boundary conditions.

**Continuum Problems:** Classification of differential equations, Variational formulation approach, Ritz method, Generalized definition of an element, Element equations from variations, Galerkin's ed residual approach, Energy balance methods.

**Element Shapes and Interpolation Functions:** Basic element shapes, Generalized co-ordinates, Polynomials, Natural co-ordinates in one- two- and three-dimensions, Lagrange and Hermite polynomials, Two-D and three-D elements for $C^0$ and $C^1$ problems, Co-ordinate transformation, Iso-parametric elements and numerical integration, Application of finite element methods to heat transfer problems.

### Course Learning Outcomes:

On successful completion of this module, students will be able to:

1. formulate simple problems into finite elements.
2. solve the elasticity and the heat transfer problems.
3. solve the complicated two- and three-dimensional problems.
4. apply finite element methods for solving real life problems arising in various fields of science and engineering.

### Text/References:


- K. J. Bathe, Finite Element Procedures, Prentice Hall, 2008.
- R. D. Cook, D. S. Malkus, M. E. Plesha, and R. J. Witt, Concepts and Applications of Finite Element Analysis, 4th Ed., John Wiley and Sons, 2001.
- J. N. Reddy, An Introduction to the Finite Element Methods, McGraw-Hill, 2006.
- E. G. Thomson, Introduction to the Finite Element: Theory, Programming and Applications, Willey, 2004.


### Evaluation Scheme:

| | |
|---|---|
| Mid-Semester Examination | 25% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes/Lab Evaluation) | 30% |

## PMA333 Introduction to Astronomy and Astrophysics

**Credits: 4 [3-2-0]**

**Course Objectives:** The goal of this course is to familiarize the students with the basic properties of astronomical objects and to provide them the knowledge of basic physics and fundamental properties that govern their structure and formation. This course also familiarizes the students with basic fluid and plasma equations useful for dynamical evolution of astrophysical systems.

**Course Outlines:**

**Introduction, Distance, Measurement system and devices:** Typical physical scales/conditions in astrophysics; order of magnitude estimation, Astronomical observations: earth vs. space based observations, Measurement systems: Distance measurements: Stellar mass measurement: Telescopes.

**Fundamentals of radiation and Sun as a star:** Radiation: geometric optics, specific intensity, luminosity/flux, radiative transfer equation, extinction and emission of light, opacity, optically thick/thin media, black body radiation, local thermal equilibrium between matter and radiation and its connection with black body radiation, Interstellar medium, Sun as a star (qualitative): Solar spectrum, effective temperature, luminosity, nuclear fusion; energy transport in the sun, X-ray emission, magnetic fields, Sunspots

**Elements of Plasma astrophysics:** Basic equations of fluid dynamics: Euler equation, continuity equation, Jeans instability, Basic equations of MHD: Flux freezing, Sunspots and magnetic buoyancy, qualitative introduction to dynamo theory, Particle acceleration in astrophysics: synchrotron radiation, Bremsstrahlung.

**Stellar structure and evolution:** Stellar models: hydrostatic equilibrium, gas/radiation pressure; theoretical main sequence, Observed stellar properties: main sequence, luminosity dependence on mass, stellar classification based on spectra, HR diagram; star clusters and distance measurements, Pre-main sequence evolution: Jeans instability, star formation, Hayashi track Post-main sequence evolution: Chandrasekhar mass limit. Type II supernova, neutronization; formation of elements heavier than iron; Neutron stars( NS); NS observed as pulsars, black hole formation for $M > NS$, Binary system evolution: effective potential in rotating frame, Lagrange points, Roche lobe, mass overflow, Type Ia supernovae, Accretion physics: magnetorotational instability.

**Galaxy and Extragalactic astronomy:** Types of galaxies: spirals, ellipticals and irregulars, Hubble pitchfork classification, Milky Way components: gas, stars, magnetic eld and cosmic rays, satellites, 21cm line, rotation cure, dark matter; HII regions, phases and components of interstellar medium, cosmic rays, Galactic dynamics: orbits in axisymmetric potentials, epicyclic limit; Oort's $A$ & $B$ constants, local differential rotation, Collisionless Boltzmann equation, Active galaxies: observations of active galaxies, quasars, unified model, radio lobes and jets; relativistic apparent superluminnal motion, Sgr $A^*$, the Galactic centre black hole, Extragalactic distance scale, structure on the largest scales.

**Course Learning Outcomes:** On successful completion of this module, students will be able to:

1. have knowledge of length scales, masses & timescales in astronomy, celestial co-ordinates and telescopes for astronomical observations
2. have basic knowledge of radiative transfer and inter-stellar medium.
3. have basic knowledge of stellar structure and their evolution, HR diagram and end states of stars.
4. have knowledge of basic components of galaxy, galactic dynamics and extragalactic sources.
5. learn the basic concepts of fluids and basic equations of MHD.

**Text/References:**

- J. Binney and S. Tremaine, Galactic Dynamics, Princeton, 2008.
- M. Harwit, Astrophysical Concepts, Springer, 2006.
- A. Rai Choudhuri, Astrophysics for Physicists, Cambridge University Press, 2012.
- J. Frank, A. King, and D. Raine, Accretion Power in Astrophysics, Cambridge University Press, 2002.
- G. B. Rybicki, A. P. Lightman, Radiative Processes in Astrophysics, Wiley, 1985.
- A. Rai Choudhuri, The Physics of Fluids and Plasmas, Cambridge University Press, 1998.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA334 Wavelet and Applications

**Credits: 4 [3-0-2]**

**Course Objectives:** The objective of this course is to cover the basic theory of wavelets, multiresolution analysis, construction of scaling functions, bases, frames and their applications in various scientific problems.

**Course Outlines:**

**Different Ways of Constructing Wavelets:** Orthonormal bases generated by a single function, The Balian- low theorem, Smooth projections on L2(R), Local sine and cosine bases and the construction of some wavelets, The unitary folding operators and the smooth projections.

**Multiresolution Analysis:** Multiresolution analysis and construction of wavelets, Construction of compactly supported wavelets and estimates for its smoothness, Band limited wavelets, Orthonormality, Completeness, First and second generation wavelet transform.

**Characterizations in the Theory of Wavelets:** Basic equations and some of its applications, Characterizations of MRA wavelets, Characterization of Lemarie-Meyer wavelets and some other characterizations, Franklin wavelets and spline wavelets on the real line, Orthonormal bases of piecewise linear continuous functions for L2(T).

**Wavelets in Signal and Image Processing:** Signals, Filters, Coding signals, Filters banks, Image analysis, Image compression.

**Laboratory Work:** Analysis of different wavelet filters, Multiresolution analysis feature of different wavelets, Applications of wavelets in signal and image processing.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. analyze the properties of various scaling functions and their wavelets.
2. analyze the properties of multiresolution analysis.
3. construct the scaling functions using infinite product formula and iterative procedure.
4. implement wavelets in various problems like image compression, denoising etc.

**Text/References:**


- H. Eugenio and W. Guido, A First Course on Wavelets, CRC Press, New York, 1996.
- C. K. Chui, An Introduction to Wavelets, Academic Press, 1992.
- Y. Meyer, Wavelets: Algorithms and Applications, SIAM, 1996.
- R. C. Gonzalez and R. E. Woods, Digital Image Processing, Pearson Education, 2007.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 25% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes/Lab Evaluation) | 30% |

## PMA335 Mathematical Biology and Nonlinear Dynamical Systems

**Credits: 4 [3-2-0]**

**Course Objectives:** The main aim of this course is to see how mathematical ideas and techniques can contribute to the understanding of questions about living things. This course will provide students with skills and knowledge in the process of the mathematical modelling cycle and enable them to formulate and specify a real-life problem.

**Mathematical Modelling in Population Biology:** Single species models, Exponential, logistic, Gompertz growth, Harvest model, Discrete time and Delay model, Interacting population model, ecological and epidemiological models, competition & mutualism models, Dynamics of exploited populations, Models for interacting populations, Reaction-diffusion equations, Age structured models, sex-structured models, models of spread, two sex models.

**Dynamical Systems:** Central manifold and Normal form, attractors, SIC, 1D map, Logistic map, Poincare maps and Poincare-Bendixson theorem, generalized Baker's map, circle map.

**Bifurcations:** Saddle-node, Transcritical, pitchfork, Hopf-bifurcation, Global bifurcations.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. construct appropriate ordinary differential equations associated with real life with relevant parameters and conditions.
2. solve the ordinary differential equations and implement equation in Matlab program to obtain numerical result.
3. analysis and stability of equilibrium of nonlinear systems in more than two variables.
4. analysis of equilibrium and stability of a reaction-diffusion equation.
5. apply Poincare-Bendixson, Central manifold and Normal theorem.

**Text/References:**


- S. H. Strogatz, Nonlinear Dynamics And Chaos: With Applications to Physics, Biology, Chemistry And Engineering, Westview Press, 2nd edition, 2014.
- L. Perko, Differential Equations and Dynamical Systems, Springer, 1996.
- J. D. Murray, Mathematical Biology I. An Introduction, 3rd Edition, Springer, 2008.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA336 Operator Theory

**Credits: 4 [3-2-0]**

**Course Objectives:** The aim of this course is to deal with the analysis of operators on a Hilbert or Banach space.

**Course Outlines:**

**Review:** Basic concepts of linear operators in normed spaces and Hilbert spaces.

**Operator Theory:** Hilbert-adjoint operator, self-adjoint, Unitary and normal operators, adjoint operator, Dual spaces, reflexive spaces, strong and weak convergence, convergence of sequence of operators and functional.

**Finite dimensional Spectral Theory of bounded self-adjoint operators:** Eigen-values and Eigen vectors, Spectrum of a bounded linear operator, spectrum of self-adjoint, positive and Unitary operators, square root of positive operator, Spectral Theorem for normal operators.

**Compact Linear Operators:** Compact Linear Operator on normed spaces, properties of compact linear operators, spectral properties of compact linear operators.

**Banach algebras:** Definitions and simple examples, Regular and singular elements. Topological divisors of zero, Spectrum of an element of Banach Algebra, formula for spectral radius.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. define strong and weak convergence in Hilbert space.
2. recognize bounded linear operators, self adjoint operators, unitary operators etc., and can prove spectral theorem.
3. understand the spectral properties of compact linear operators.
4. describe Banach algebras and its properties.

**Text/References:**


- G. F. Simmons, Introduction to Topology and Modern Analysis, Mc Graw-Hill, 1963.
- E. Kreyszig, Introduction to Functional Analysis with Applications, John Wiley & Sons, 1978.
- B. V. Limaye, Functional Analysis, New Age International Limited, 1996.
- P. K. Jain, O. P. Ahuja, and K. Ahmed, Functional Analysis, New Age International Ltd., 1995.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA337 Enumerative Combinatorics

**Credits: 4 [3-2-0]**

**Course Objectives:** The objective of this course is to introduce the fundamentals of mathematical structures that are discrete, dealing with integral solutions, enumeration and counting problems. It has applications in other mathematical courses such as partition theory, coding theory, combinatorial optimization, or designs problem and in all fields of computer science especially in data structure algorithms and graph theory.

**Course Outlines:**

**Counting Principles:** Two basic counting principles, permutation and combinations, sequences and selections, injection and bijection principles, distribution problems.

**Binomial coefficients and Principle of Inclusion-Exclusion:** Binomial Coefficients, Pascal's Triangle, unimodality of binomial coefficient, multinomial coefficients, and associated properties, Pigeonhole principle and principle of inclusion-exclusion.

**Generating functions:** Ordinary generating functions, exponential generating functions generating permutations, generating combinations, Partitions of Integers, Ferrers graph, Compositions, zig-zag graphs.

**Recurrence Relations:** Linear Homogeneous Recurrence relations, General Linear recurrence relation, system of linear recurrence relation, solving recurrence relations using generating function, non-linear recurrence relations, Catalan Numbers, Stirling Numbers, Lattice Paths and Schroder Numbers.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. solve basic counting problems involving number of permutations, number of combinations and principle of inclusion-exclusion.
2. manipulate and derive properties of formal power series via employing properties of binomial coefficient and multinomial coefficient.
3. derive recurrence relations, generating functions and provide explicit formulas for various combinatorial sequences.
4. construct combinatorial proofs of identities and inequalities using generating functions and binomial coefficients.

**Text/References:**


- R. A. Brnaldi, Introductory Combinatorics, Pearson, 5th ed., 2009.
- C. Chuan-Chong, K. Khee-Meng, Principles and Techniques in Combinatorics, World Scientific, 2010.
- G.E. Martin, Counting: The Art of Enumerative Combinatorics, Springer, 2001.
- A. Tuker, Applied Combinatorics, Wiley, 2017.
- R. P. Grimaldi, B. V. Ramana, Discrete and Combinatorial mathematics, Pearson, 2007.
- Richard P. Stanley, Enumerative Combinatorics, Vol 1, 2nd edition, Cambridge University Press, 2011.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA338 Fuzzy Sets and Applications

**Credits: 4 [3-2-0]**

**Course Objectives:** The objective of this course is to teach the students the need of fuzzy sets, arithmetic operations on fuzzy sets, fuzzy relations, possibility theory, fuzzy logic, and its applications.

**Course Outlines:**

**Classical and Fuzzy Sets:** Overview of classical sets, Membership function, $\alpha$-cuts, Properties of $\alpha$-cuts, Extension principle.

**Operations on Fuzzy Sets:** Compliment, Intersections, Unions, Combinations of operations, Aggregation operations.

**Fuzzy Arithmetic:** Fuzzy numbers, Linguistic variables, Arithmetic operations on intervals and numbers, Fuzzy equations.

**Fuzzy Relations:** Crisp and fuzzy relations, Projections and cylindric extensions, Binary fuzzy relations, Binary relations on single set, Equivalence, Compatibility and ordering Relations, Morphisms, Fuzzy relation equations.

**Possibility Theory:** Fuzzy measures, Evidence and possibility theory, Possibility versus probability theory.

**Fuzzy Logic:** Classical logic, Multivalued logics, Fuzzy propositions, Fuzzy qualifiers, Linguistic hedges.

**Applications of Fuzzy Logic :** Control systems engineering, Power engineering and Optimization.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. construct the appropriate fuzzy numbers corresponding to uncertain and imprecise collected data.
2. explain the problems having uncertain and imprecise data.
3. find the optimal solution of mathematical programming problems having uncertain and imprecise data.
4. deal with the fuzzy logic problems in real world problems.

**Text/References:**


- G. J. Klir and T. A. Folger, Fuzzy Sets, Uncertainty and Information, Prentice Hall, 1988.
- G. J. Klir and B. Yuan, Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall, 1995.
- H. J. Zimmermann, Fuzzy Set Theory and its Applications, Allied Publishers, 1991.
- C. Mohan, An introduction to Fuzzy Set Theory and Fuzzy Logic, Viva Publishers, 2009.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA339 Fluid Mechanics

**Credits: 4 [3-2-0]**

**Course Objectives:** This course is intended to provide a treatment of topics in fluid mechanics to a standard where the student will be able to apply the techniques used in deriving a range of important results and in research problems. The objective is to provide the student with knowledge of the fundamentals of fluid mechanics and an appreciation of their application to real world problems.

**Course Outlines:**

**Kinematics:** Lagrangian and Eulerian methods, Equation of continuity, Stream lines, Path lines and streak lines, Velocity potential and stream function, Irrotational and rotational motions.

**Dynamics:** Euler's equation, Bernoulli's equation, Equations referred to moving axes, Impulsive actions, Vortex motion and its elementary properties, Motions due to circular and rectilinear vortices, Kelvins proof of permanence.

**Potential Flow:** Irrotational motion in two-dimensions, Complex-velocity potential sources, Sinks, Doublets and their images, Conformal mapping.

**Laminar Flow:** Stress components in a real fluid, Navier-Stokes equations of motion, Plane poiseiuille and couette flows between two parallel plates, Flow through a pipe of uniform cross section in the form of circle, Annulus, Theory of lubrication.

**Boundary Layer Flows:** Boundary layer thickness, Displacement thickness, Prandit's boundary layer, Boundary layer equations in two dimensions, Blasius solution, Karman integral equation, Separation of boundary layer flow.

**Learning Outcomes:**

On successful completion of this module, students will be able to:

1. describe the basic principles of fluid mechanics, such as Lagrangian and Eulerian approach, conservation of mass etc.
2. apply Euler and Bernoulli's equations and the conservation of mass to determine velocity and acceleration for incompressible and inviscid fluid.
3. explain the concept of rotational and irrotational flow, stream functions, velocity potential, sink, source, vortex etc.
4. analyse simple fluid flow problems (flow between parallel plates, flow through pipe etc.) with Navier - Stoke's equation of motion.
5. analyse the phenomenon of flow separation and boundary layer theory.

**Text/References:**


- S. W. Yuan, Foundations of Fluid Mechanics, Prentice Hall, 1976.
- F. Chorlton, Textbook of Fluid Dynamics, C.B.S. Publishers, 2005.
- W. H. Besant and A. S. Ramsay, Treatise of Hydro Mechanics, Part II, CBS Publishers, 2004.
- R. K. Rathy, An Introduction to fluid Dynamics, Oxford and IBH Publishing Company, 1976.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA340 Discrete Mathematical Structures

**Credits: 4 [3-2-0]**

**Course Objectives:** This course develop mathematical foundations for explaining and creating mathematical arguments, solving problems by constructing recurrence relations and generating functions, and exploring counting problems that are often required in learning other mathematical and computer sciences courses.

**Course Outlines:**

**Mathematical Logic:** Statement and notations, Connectives, Statement formulas and truth table, Conditional and bi-conditional statements, Tautology and contradiction, Equivalence of formulas, Tautological implications.

**Theory of inference:** Validity using truth table, Rules of inference, Consistency of premises and indirect method of proof, Predicates, Statement function, Variables, Quantifiers, Free and bound variables, Universe of discourse, Inference of the predicate calculus.

**Relation:** Review of binary relations, equivalence relations, counting of reflexive and symmetric relations, Compatibility relation, Composition of binary relations, Composition of binary relations and transitive closure, Partial ordering and partial ordered set.

**Function:** Review of functions and their enumeration, Pigeonhole principle.

**Recurrence Relation:** Iteration, Recurrence relations, Generating function, solving linear recurrence relation, solving recurrence relation using generating function.

**Lattice and Boolean Algebra:** Lattice and algebraic system, Basic properties of algebraic systems, Special types of lattices, Distributed, Complemented lattices, Boolean algebra, Boolean expressions, Normal form of Boolean expressions, Boolean function and its applications to LOGIC GATES.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. construct mathematical arguments using logical connectives and quantifiers.
2. validate the correctness of an argument using statement and predicate calculus.
3. apply lattices and Boolean algebra as tools in the study of network.
4. explain some of the discrete structures which include sets, relations, functions, and graphs.
5. construct and solve recurrence relations with or without generating functions.

**Text/References:**


- J. P. Tremblay and R. Manohar, A First Course in Discrete Structures with Applications to Computer Science, McGraw Hill, 1987.
- Kenneth H. Rosen, Discrete Mathematics and its Applications, McGraw Hill Education; 7th edition, 2017.
- C. L. Liu, Elements of Discrete Mathematics, McGraw Hill, New York, 1978.
- R. P. Grimaldi and B. V. Ramana, Discrete and Combinatorial Mathematics – An Applied Introduction, Pearson Education, 5th edition, 2006.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

# Elective II

## PMA432 Modeling of Stellar Structure

**Credits: 4 [3-2-0]**

**Course Objectives:** The goal of this course is to familiarize the students with the basic observed properties of stars and to provide them the knowledge of basic physics and fundamental properties that govern stars and their structures. The aim of this course is also to explain mathematical techniques to solve the stellar structure equations and apply the basic theory of stellar structures on analytical models.

**Course Outlines:**

**Observed Properties of the Stars:** Introduction to stars, Measurement of stellar distances, Luminosities, Temperatures, Masses and radii, The Hertzsprung-Russell diagram.

**Fundamental Equations of Stellar Structure:** Time scales, Fundamental equations: Mass conservation, Hydrostatic equilibrium, Energy transport, The virial theorem. Radiative transport and convection.

**Boundary Value Problems:** Shooting method and relaxation method, Applications to stellar structure with detailed discussion of Henyey scheme and EZ-Code.

**Stellar Modelling and Numerical Calculations:** Russell-Voigt theorem, Limits to the mass, solving the coupled equations, Simple analytic stellar models: Polytropes and other relations, Numerical models, The Edington luminosity, Dimensional analysis and mass-radius relations, The HR diagram.

**Superdense Objects:** Use of polytropic models for completely degenerate stars, Mass-radius relation, Non-degenerate upper layers and abundance of Hydrogen, Stability of white dwarfs.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. describe the various properties of stars.
2. obtain the basic physics and fundamental properties that govern star and their structure.
3. apply the mathematical methods to solve stellar structure equations.
4. develop mathematical methods of stellar structures and their solution techniques.
5. explain the various properties of super dense objects like white dwarf stars.

**Text/References:**


- S. Chandrasekhar, An introduction to the Study of Stellar Structure, University of Chicago Press, Reprinted by Dover, 1939.
- R. Kippenhahn and A. Weigert, Stellar Structure and Evolution, Springer-Verlag, 1990.
- M. Schwarzschild, Structure and Evolution of Stars, Princeton University Press, 1958.
- D. Prialnik, An Introduction to the Theory of Stellar Structure and Evolution, CUP, 2000.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA433 Asymptotic Methods and Perturbation Theory

**Credits: 4 [3-2-0]**

**Course Objectives:** The course aims to introduce asymptotic methods and perturbation theory to graduate students which have wide applications in solving linear and nonlinear differential equations especially those which model multi scale phenomena.
Prerequisites: Undergraduate level knowledge of calculus and differential equations, and basic complex analysis.

**Course Outlines:**

**Review of elementary differential equations:** Introduction to differential equations in the complex plane (Painleve equations).

**Approximate solutions to linear differential equations:** Classification of singular points of homogeneous linear differential equations, local series expansion about regular singular points (Taylor and Fuch's series), local solutions around irregular singular points (method of dominant balance, basic ideas of asymptotic series expansion).

**Approximate solutions to nonlinear differential equations:** Spontaneous singularities, approximate solutions (Painleve transcendent, Thomas-Fermi equation, phase-space interpretation of nonlinear autonomous systems, classification of critical points).

**Asymptotic expansion of integrals:** Integration by parts, Laplace's method and Watson's lemma, method of stationary phase, method of steepest descents.

**Perturbation theory:** Regular and singular perturbation theory, asymptotic matching (method of matched asymptotic expansions to solve differential equations), basic ideas of boundary layer theory, WKB approximation to solve differential equations with dissipative and dispersive behaviour, examples of the one-Turning-point problem and tunneling via the Schrodinger equation.

**Multiple scales analysis:** Resonance and secular behaviour, Fredholm alternative, examples of multiple scales analysis through models of damped oscillators, Duffing equation and elementary ideas of stability.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. find approximate solutions of differential equations as power series (asymptotic series) around different singular points.
2. compute approximate solutions to integrals.
3. find approximate solutions of differential equations using perturbation theory.

**Text/References:**


- C. M. Bender and S. A. Orszag, Advanced Mathematical Methods for Scientists and Engineers, Springer, 1999.
- E. J. Hinch, Perturbative Methods, Cambridge University Press, 1991.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA434 Theory of Elasticity

**Credits: 4 [3-2-0]**

**Course Objectives:** This course is intended to provide a basic treatment of the formulation of linear elasticity theory and its application to problems of stress and displacement analysis. The objective is to provide the student knowledge of fundamentals of theory of elasticity and an appreciation of their application to the different fields of research.

**Course Outlines:**

**Tensor Algebra:** Scalar, Vector, Matrix and tensor definition, Index notation, Kroneckor delta and alternating symbol, Coordinate-transformation, Cartesian tensor of different order, Properties of tensors, Isotropic tensors of different orders and relation between them, Symmetric and skew-symmetric tensors, Covariant, Contra variant and mixed tensors, Sum and product of tensors.

**Analysis of Stress:** Stress vector, Stress components, Stress tensor, Symmetry of stress tensor, Stress quadric of Cauchy, Principal stress and invariants, Maximum normal and shear stresses.

**Analysis of Strain:** Affine transformations, Infinitesimal affine deformation, Geometrical interpretation of the components of strain, Strain quadric of Cauchy, Principal strains and invariants, General infinitesimal deformation, Finite deformations, Examples of uniform dilatation, Simple extension and shearing strain.

**Equations of Elasticity:** Generalized Hooke's law, Hooke's law for Homogeneous isotropic media, Elastic moduli for isotropic media, Equilibrium and dynamic equations for an isotropic elastic solid, Beltrami-Michell compatibility equations, Strain energy function.

**Elastic Waves:** Simple harmonic progressive waves, Scalar wave equation, Progressive type solutions, Plane waves, Propagation of waves in an unbounded elastic solid media, P, SV and SH waves, Elastic surface waves as Rayleigh waves, Love waves. Applications to different elastic models.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. describe the notation and properties of different types of tensor.
2. explain various terms related to stress tensor like normal and shear stress, stress quadric of cauchy, Principal stress and invariants.
3. explain affine transformations and geometrical interpretation of the components of strain and terms related to strain tensor.
4. apply the generalized Hookes law, reduction of elastic constants to different elastic models from the most general case.
5. develop equilibrium and dynamical equations of an isotropic elastic solid.
6. obtain some important aspects of wave propagation in the infinite and semi-infinite solids.

**Text/References:**


- I. S. Sokolnikoff, Mathematical Theory of Elasticity, Tata McGraw Hill, New Delhi, 1977.
- D. S. Chandrasekharaiah, and L. Debnath, Continuum Mechanics, Academic Press, 1990.
- K. F. Graff, Wave Motion in Elastic Solids, Dover, New York, 1991.
- A. A. Shaikh, U. C. De. and J. Sengupta, Tensor Calculus, Narosa Publishing House, 2nd ed., 2008.
- S. Narayan, Text Book of Cartesian Tensor, S. Chand & Co., 1956.

### Evaluation Scheme:

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA435 Algebraic Coding Theory

**Credits: 4 [3-2-0]**

**Course Objectives:** The objective of the course is to introduce basic topics of algebraic coding theory like error correction and detection, linear codes, Hamming codes, finite fields and BCH codes, dual codes and the distribution, cyclic codes, generator polynomial and check polynomial.

**Course Outlines:**

**Introduction to Coding Theory:** Code words, Distance and function, Nearest-neighbour decoding principle, Error detection and correction, Matrix encoding techniques, Matrix codes, Group codes, Decoding by coset leaders, Generator and parity check matrices, Syndrom decoding procedure, Dual codes.

**Linear Codes:** Linear codes, Matrix description of linear codes, Equivalence of linear codes, Minimum distance of linear codes, Dual code of a linear code, distribution of the dual code of a binary linear code, Hamming codes.

**BCH Codes:** Polynomial codes, Finite fields, Minimal and primitive polynomials, Bose-Chaudhuri-Hocquenghem codes.

**Cyclic Codes:** Cyclic codes, Algebraic description of cyclic codes, Check polynomial, BCH and Hamming codes as cyclic codes.

**MDS Codes:** Maximum distance separable codes, Necessary and sufficient conditions for MDS codes, distribution of MDS codes.

**Algebraic Coding Theory:** Overview of coding theory, Error detecting and correcting codes.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. apply basic techniques of algebraic coding theory like matrix encoding, polynomial encoding, and decoding by coset leaders etc.
2. analyze different types of codes like linear, BCH, cyclic and MDS codes.
3. apply algebraic coding theory is applicable to solve the real world problems.

**Text/References:**


- L. R. Vermani, Elements of Algebraic Coding Theory, Chapman and Hall, 1996.
- P. Vera, Introduction to the Theory of Error Correcting Codes, John Wiley and Sons, 1998.
- S. Roman, Coding and Information Theory, Springer Verlag, 1992.
- P. Garrett, The Mathematics of Coding Theory, Pearson Education, 2004.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA436 Topological Vector Spaces

**Credits:4 [3-2-0]**

**Course Objectives:** The aim of this course is to deal with topological vector spaces with their properties.

**Course Outlines:**

Definition and examples of topological vector spaces, Convex, balanced and absorbing sets and their properties.

Minkowski's functional, Subspace, product space and quotient space of a topological vector space.

Locally convex topological vector spaces, Normable and metrizable topological vector spaces, Complete topological vector spaces and Frechet space.

Linear transformations and linear functional and their continuity, Finite dimensional topological vector spaces, Hahn-Banach Theorem.

Uniform bounded Principle, Open mapping theorem and closed graph theorem for Frechet spaces.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. define topological vector spaces and their properties.
2. recognize subspace, product, quotient space, normable and metrizable topological vector spaces.
3. define linear transformation and related theorems.

**Text/References:**


- W. Rudin, Functional Analysis, McGraw Hill Education, 2nd edition, 2017.
- H. H. Schaefer, Topological Vector Spaces, Springer, 1971.
- J. Horvath, Topological Vector Spaces and Distributions, Addison-Wesley, 1966.
- G. Kothe, Topological Vector Spaces , Vol. I, Springer, New York, 1969.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA437 Fixed Point Theory

**Credits: 4 [3-2-0]**

**Course Objectives:** The aim of this course is to deal with fixed point theory with their applications.

**Course Outlines:**

**Introductory Concepts:** Topological Preliminaries, Metric Spaces, Normed Spaces, Inner Product Spaces, Topological Vector Spaces, Locally Convex Spaces, Normal Structure.

**Metric Fixed Point Theory:** Fixed Points, Lipschitz Mapping, Contraction mapping, The Banach Contraction Principle, Fixed Point Theorems for Nonexpansive Mappings, Quasi-nonexpansive Mappings and Fixed Points. Application to System of Linear Equations, Differential Equations and Integral Equations.

**Topological Fixed Point Theory:** Topological Fixed point property, retraction, multivalued mappings, upper semi-continuous, lower semi-continuous, Sperner's lemma, Brouwer Fixed point theorem, Knaster, Kuratoski and Mazurkiewicz (KKM) theorem, Schauder Fixed Point theorem, Tychonoff Theorem.

**Order Theoretic Fixed Point Theory:** Partially ordered set, conditionally complete, countably chain complete POSETS, Tarski-Kantorovitch fixed point theorem, Tarski fixed point theorem, Knaster-Tarski theorem.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. discuss the existence and uniqueness of fixed points using contraction mapping principle.
2. discuss the existence of fixed points using topological aspects.
3. discuss the existence of fixed points for partially ordered sets.
4. find the solution of nonlinear real-world problem using fixed point approach.

**Text/References:**


- P. V. Subramanyam, Elementary Fixed Point Theorems, Springer, 2018.
- R. P. Agarwal, M. Meehan and D. O'Regan, Fixed Point Theory and Applications, Cambridge University Press, 2004.
- K. Goebel and W. A. Kirk, Topics in Metric Fixed Point Theory, Cambridge University Press, 1990.
- V. I. Istratescu, Fixed Point Theory: An Introduction, Springer, 2001.
- M. A. Khamsi and W. A. Kirk, An Introduction to Metric Spaces and Fixed Point Theory, John Wiley & Sons, 2001.
- E. Zeidler, Nonlinear Functional Analysis and its Applications I: Fixed-Point Theorems, Springer, 1986.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA438 Statistical Simulation and Computation

**Credits: 4 [3-0-2]**

**Course Objectives:** This course introduces statistical simulation. Students learn Pattern Recognition using statistical concepts.

**Course Outlines:**

**Simulation:** Introduction, Systems, Models, types of models, need of simulation, Monte Carlo method, physical versus digital simulation, Buffen's needle problem.

**Random Number Generation:** Mid square method, Congruential generators, Shift generator, statistical tests for pseudo random numbers.

**Pattern Recognition:** Introduction, Basic Concepts, Fundamental problems, Design concepts and methodologies, Examples of automatic systems, Pattern Recognition model, and Pattern classification by likelihood functions.

**Random Variate Generation:** Inverse transformation method, Acceptance-Rejection method, Composition method. Simulation of Random vectors, Multivariate transformation method, Generation from Multinormal distribution, Generating random variates from continuous distributions.

**Monte Carlo integration:** Hit or miss Monte Carlo method,sample mean Monte Carlo method, Efficiency of Monte Carlo method.

**Variance Reduction Techniques:** Introduction, Importance sampling, Correlated sampling, Control variates, Stratified sampling.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. apply the fundamental knowledge of statistical simulation and computation.
2. apply Monte Carlo and other types of numerical methods for analyzing complex models where the simple numerical methods cannot be applied.
3. understand the Mid square method, Congruential generator, shift generator etc. for pseudo random numbers.
4. evaluate Monte Carlo integration.

**Text/References:**


- Reuven Y. Rubinstein, Dirk P. Kroese, Simulation and the Monte Carlo Method, 3rd Edition, Willey, 2016.
- Frank L. Severance, System Modeling and Simulation: An Introduction, Willey, 2001.
- W. A. Lewis and Ed McKenzie, Simulation Methodology for Statisticians, Operations Analysts, and Engineers, Chapman and Hall/CRC, 1989.
- T. T. Julius and R. C. Gonzalesz, Pattern Recognition Principles, Addison-Wesley, 1977.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 25% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes/Lab Evaluation) | 30% |

# PMA439 Financial Mathematics

**Credits: 4 [3-0-2]**

**Course Objectives:** This is an introductory course in finance to equip with a framework and basic techniques necessary for financial engineering. The main focus is on valuation of financial assets and more specifically derivative products. The course will introduce the concept of risk and relation between risk and return. The knowledge of risk and valuation will be integrated in optimal decision-making. The models will be studied in discrete-time scenario.

**Course Outlines:**

**Basics of Financial Mathematics:** Financial markets, terminologies, basic definitions and assumptions, Interest rate, present value, future value, NPV, annuity and perpetuity. Market structure, no arbitrage principle, derivative products, forwards, futures their valuation, dividend and non divided cases, options, swap, valuation concept, purpose and working of these products.

**Theory of Option Pricing:** Options-calls and puts, pay-off, profit diagrams, hedging and speculation properties of options, valuation of options using pricing and replication strate- gies, mathematical properties of their value functions, put-call parity. Risk neutral probability measure (RNPM) (discrete case), existence of RNPM, Binomial lattice model, Binomial formula for pricing European style and American style options, dividend and non-divided cases. CRR model, Black-Scholes formula derivation, Examples. Greeks and their role in hedging, delta-neutral portfolio, delta-gamma neutral portfolio.

**Portfolio Optimization:** Portfolio optimization: introduction, risk, return, two-assets portfolio, Markowitz curve, efficient frontier, Multi-assets all risky portfolio, mean-variance Markowitz model, two fund theorem. Portfolio with one risk free asset, one fund theorem, CAPM, market line, beta, systematic and unsystematic risks, factor models, other risk measures, stochastic dominance and portfolio optimization. Risk neutral probability measure (RNPM) (discrete case), existence of RNPM, Binomial lattice model, Binomial formula for pricing both European style and American style options, dividend and non-divided cases.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. understand basic quantities that are reported in everyday life such as interest rates, periodic payments of money, dividends, shares, bonds, forwards, futures etc.
2. evaluate call and put option prices using binomial and CRR models.
3. construct a portfolio which is optimal in a given market scenario.

**Text/References:**


- D.G. Luenberger, Investment Science, Oxford University Press, 2014.
- S. Chandra, S. Dharmaraja, A. Mehra, R. Khemchandani, Financial Mathematics: An Introduction, Narosa, 2014.
- M. Capinsky and T. Zastawniak, Mathematics for Finance: An Introduction to Financial Engineering, Springer, 2010.
- J. C. Hull, Options, Futures and other Derivatives, Prentice Hall, 10th edn, 2018.
- J. H. Cochrane, Asset Pricing, Princeton University Press, 2005.

**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 25% |
| End-Semester Examination | 45% |
| Sessionals (Assignments/Quizzes/Lab Evaluation) | 30% |

## PMA440 Combinatorial Programming

**Credits: 4 [3-2-0]**

**Course Objectives:** The course is a comprehensive introduction to the theory, algorithms and applications of integer optimization and is organized in four parts: formulations and relaxations, algebra and geometry of integer optimization, algorithms for integer optimization, and extensions of integer optimization.

**Course Outlines:**
Polyhedral Combinatorics (1) Basic polyhedral theory (2) Linear Programming: Quick overview of duality, algorithms for LP - Equivalence of optimization and separation (3) Integer Programming : - Integer hull of a polyhedron - Cutting plane algorithms and bounds - Branch and bound, branch and cut algorithms - Totally unimodular matrices (TUM), Total Dual Integrality (TDI).
Combinatorial algorithms for classic discrete optimization problems
(1) Quick Overview of flow problems: Maximum flow, Minimum Cut, Minimum cost flow, Multicommodity flows
(2) Matching theory: Matching and alternating paths
(3) Set covering.
Other techniques for Combinatorial Optimization
(1) Matroid Theory, Greedy Algorithms
(2) Lattice, Lattice basis reduction, Lenstra's algorithm.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. formulate and analyze mathematical models of some classical combinatorial optimization problems.
2. implement cutting plane algorithms to integer programs.
3. use various algorithms to solve network flow problems.

**Text/References:**


- G. L. Nemhauser and L.A. Wolsey, Integer and Combinatorial Optimization, John Wiley, 1988.
- W. J. Cook, W. H. Cunningham, W. R. Pulleyblank, and A. Schrijver, Combinatorial Optimization, John Wiley and Sons, 1998.
- A. Schrijver, Combinatorial Optimization-Polyhedra and Efficiency, Springer-Verlag, 2003.
- M. Conforti, G. Cornuejols and G. Zambell, Integer Programming, Springer, 2014.
- R. K. Ahuja, T. L. Manganti and J. B. Orlin, Network Flows: Theory, Applications and Algorithms, Prentice Hall, 1993.
- L. R. Foulds, Combinatorial Optimization for Undergraduates, UTM, Springer Verlag, 1984.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA441 Stochastic Calculus

**Credits: 4 [3-2-0]**

**Course Objectives:** This course explanations and expositions of stochastic processes concepts which they need for their experiments and research. It also covers theoretical concepts pertaining to handling various stochastic modeling. This course provides classification and properties of stochastic processes, stationary processes, discrete and continuous time Markov chains and simple Markovian queueing models.

**Course Outlines:**

**Probability Review and Introduction to Stochastic Processes:** Probability spaces, random variables and probability distributions, transforms and generating functions, convergence, Definition, examples and classification of random processes according to state space and parameter space, Stochastic processes, filtrations and stopping times.

**Martingales:** Conditional expectations, definition and examples of martingales, applications in finance.

**Brownian Motion:** Brownian motion in one and several dimensions, basic properties, functionals and sample path properties.

**Ito stochastic integration:** Basic definitions, elementary properties, characterizations, Ito's rule (change-of-variable formulas), and applications of Ito's rule to results such as the martingale characterization of Brownian motion and to martingale moment inequalities.

**Stochastic equation:** Stochastic differential equation, Stochastic integral equation, Ito formula, some important SDEs and their solutions, applications to finance.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. give examples and classification of random processes and stationary processes.
2. provide examples and applications of martingales.
3. give examples and application of Brownian motion.
4. apply Ito's formula for integration and can solve differential as well as integral equations.

**Text/References:**


- J. Michael Steele, Stochastic Calculus and Financial Applications, Springer, 2010.
- J. Medhi, Stochastic Processes, 3rd Edition, New Age International, 2009.
- S.M. Ross, Stochastic Processes, 2nd Edition, Wiley, 1996.
- S. E. Shreve, Stochastic Calculus for Finance, Vol. I & Vol. II, Springer, 2004.


**Evaluation Scheme:**

| | |
|---|---|
| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Assignments/Quizzes etc.) | 20% |

## PMA442 Advanced Numerical Optimization Techniques

**Credits: 4 [3-2-0]**

**Course Objectives:** This course aims to introduce students to use constrained and unconstrained optimization problems and applications.

**Course Outlines:**

**Introduction:** Review of the basic concepts of convex functions and Linear programming theory and KKT conditions.

**Unconstrained optimization methods:** Line search, trust region methods, gradient descent, conjugate gradient, Newton and Quasi-Newton methods, Davidon-Fletcher-Powell (DFP) method, least square problems.

**Constrained Optimization methods:** Frank and Wolfe's method, Rosen's gradient projection method, penalty function method, barrier function method and interior point method.

**Semi-definite programming:** Formulation of semi-definite programming problems and applications. Formulations of dual problems and duality theorems.

**Nonlinear programming techniques:** Separable programming, Linear fractional programming and Complementary pivoting algorithm.

**Course Learning Outcomes:**

On successful completion of this module, students will be able to:

1. use various techniques to solve unconstrained optimization problems.
2. solve constrained optimization problem by using Newton method and its variants.
3. solve semi-definite programming problems.
4. apply vertex search algorithms to nonlinear programming problems.

**Text/References:**


- S. Chandra, Jayadeva, and A. Mehra, Numerical Optimization and Applications, Narosa Publishing House, 2013.
- R. W. Cottle and M. N. Thapa, Linear and Nonlinear Optimization, Springer, 2017.
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**Evaluation Scheme:**

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| Mid-Semester Examination | 30% |
| End-Semester Examination | 50% |
| Sessionals (Quizzes/Assignments etc.) | 20% |